The Mixed States of Associative Memories Realize Unimodal Distribution of Dominance Durations in Multistable Perception

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Abstract. We propose a pulse neural network that exhibits chaotic pattern alternations among stored patterns as a model of multistable perception, which is reflected in phenomena such as binocular rivalry and perceptual ambiguity. When we regard the mixed state of patterns as a part of each pattern, the durations of the retrieved pattern obey unimodal distributions. The mixed states of the patterns are essential to obtain the results that are consistent with psychological studies. Based on these results, it is proposed that many pre-existing attractors in the brain might relate to the general category of multistable phenomena, such as binocular rivalry and perceptual ambiguity.

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1 Introduction

In the perception of visual information, it is well known that multiple stable states compete for perceptual dominance. For example, when two different stimuli are presented to the eyes, the dominant stimulus perceived fluctuates over time, a phenomenon known as binocular rivalry [1, 2]. Similarly, when an ambiguous figure such as a Necker cube is presented, the dominant interpretation also fluctuates over time [3]. Research has also indicated that the duration of the dominant state (dominance duration) may be characterized by a unimodal distribution, such as the gamma distribution [2, 3] or the log-normal distribution [1].

One possible mechanism for such fluctuations in multistable perception is associated with noise in the visual system, which is generated by small eye movements and microsaccades. On the other hand, the deterministic chaos generated by nonlinear dynamics in the brain may also be responsible for such fluctuations. Several dynamical models in which the state of the network changes chaotically among several patterns have been proposed [4–6]. However, the duration of a pattern in the chaotic networks does not obey a unimodal distribution, but it typically obeys a monotonically decreasing distribution [6].

In the present study, we report that the pattern alternations caused by chaotic dynamics of a pulse neural network can reproduce the properties of multistable perception. This network is composed of neuronal models which emit pulses when a sufficiently strong input is injected [7–9], while the previous models were composed of conventional neuronal models based on firing rates. By storing several patterns based on the mechanism of associative memory, this network shows chaotic pattern alternations [10, 11]. It is observed that the durations of the retrieved pattern obey unimodal distributions when we regard the mixed state of patterns as a part of each pattern.

Based on these results, it is proposed that many pre-existing attractors in the brain might relate to the general category of multistable phenomena, such as binocular rivalry and perceptual ambiguity. In the previous work, we called such a set of pre-existing attractors as “attractor landscape” [11].

This paper is organized as follows. In section 2, we define a pulse neural network composed of excitatory neurons and inhibitory neurons exhibiting synchronized, chaotic firing. This network is referred to as the one-module system. In section 3, we connect eight modules of networks in which three patterns are stored according to the mechanism of associative memory. We show that chaotic dynamics are responsible for alterations in the retrieved patterns over time. It is observed that the durations of the retrieved pattern are shown to obey unimodal distributions. The final section provides conclusions.

2 One-module system

In sections 2 and 3, we introduce a neural network of theta neurons with phases as their internal states [7–9]. When a sufficiently strong input is provided, each neuron yields a pulse by increasing its phase around a circle and returning to its original phase. The network is composed of N_E excitatory neurons and N_I inhibitory neurons governed by the following equations:

$$\dot{\theta}_E^{(i)} = (1-\cos\theta_E^{(i)}) + (1+\cos\theta_E^{(i)})(r + \xi_E^{(i)}(t) + g_{int}I_E(t) - g_{ext}I_I(t)), \quad (1)$$

$$\dot{\theta}_I^{(i)} = (1-\cos\theta_I^{(i)}) + (1+\cos\theta_I^{(i)})(r + \xi_I^{(i)}(t) + g_{ext}I_E(t) - g_{int}I_I(t)), \quad (2)$$

$$I_X(t) = \frac{1}{2N_X}\sum_{j=1}^{N_X}\sum_k \frac{1}{\kappa_X}\exp\left(-\frac{t-t_k^{(j)}}{\kappa_X}\right), \quad (3)$$

$$\langle \xi_X^{(i)}(t)\xi_Y^{(j)}(t')\rangle = D\delta_{XY}\delta_{ij}\delta(t-t'), \quad (4)$$

where $\theta_E^{(i)}$ and $\theta_I^{(i)}$ are the phases of the ith excitatory neuron and the ith inhibitory neuron, respectively. r is a parameter of the neurons that determines whether the equilibrium of each neuron is stable or not. We used $r = -0.025$ to ensure that each neuron had a stable equilibrium. $X = E$ or I denote the excitatory or inhibitory ensemble, respectively, while $t_k^{(j)}$ is the kth firing time of the jth neuron in the ensemble X, and the firing time is defined as the time at which $\theta_X^{(j)}$ exceeds π in the positive direction. The neurons communicate with each other using the post-synaptic potentials whose waveforms are the exponential functions as shown in Eq.(3). $\xi_X^{(i)}(t)$ represents Gaussian white noise added to the ith neuron in the ensemble X.

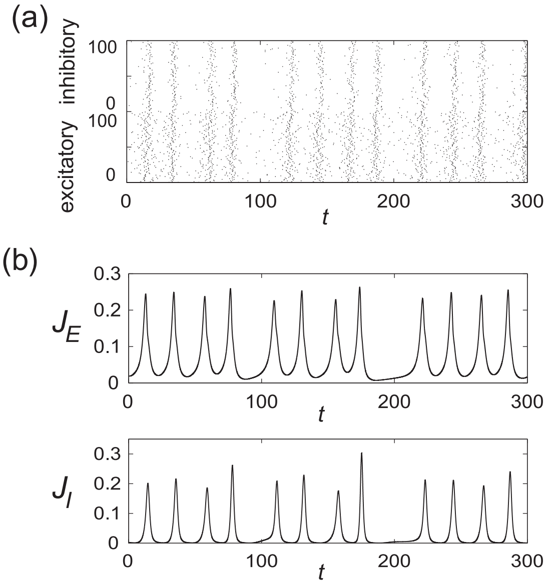

Fig. 1. (a) Chaotic synchronization observed in a module with $D = 0.0032$, $r = -0.025$, $g_{int} = 4$, and $g_{ext} = 2.5$. Raster plot of spikes of 200 randomly chosen excitatory neurons and inhibitory neurons in a module with $N_E = N_I = 2000$ is shown. (b) Chaotic synchronization in a module with an infinite number of neurons obtained by analysis with Fokker-Planck equations. The values of parameters are the same as those used in (a). Temporal changes in the instantaneous firing rates J_E and J_I are shown.

In the following, this network is referred to as a one-module system, which exhibits various patterns of synchronized firing [9]. We utilized the chaotic synchronization shown in Fig. 1. In Fig. 1(a), a raster plot of spikes of 200 randomly chosen excitatory neurons and inhibitory neurons in a module with $N_E = N_I = 2000$ is shown. This plot allows one to observe the synchronized firing of neurons, and that the intervals of synchronized firing do not remain constant. To analyze this variability, we took the limit of $N_E, N_I \to \infty$ in order to obtain the Fokker-Planck equation, which governs the dynamics of the probability densities $n_E(\theta_E)$ and $n_I(\theta_I)$ of $\theta_E^{(i)}$ and $\theta_I^{(i)}$ as shown in Ref.[10]. The instantaneous firing rates J_E and J_I of the excitatory and inhibitory ensembles obtained from the analysis of the Fokker-Planck equation are shown in Fig. 1(b). The largest Lyapunov exponent of the data in Fig. 1(b) is positive [9], indicating that the dynamics of J_E and J_I are chaotic.

In the following, only the one-module systems with infinite neurons treated in Fig. 1(b) are considered, as the Fokker-Planck equation does not contain noise, allowing for the reproduction of analyses.

3 Pattern alternations in multiple modules of network

In this section, we define a network with multiple modules [10, 11]. Several patterns can be stored in this network according to the mechanism of associative memory.

The synaptic inputs T_{Ei} and T_{Ii} injected to the ith excitatory ensemble Ei and the inhibitory ensemble Ii, respectively, are defined as

$$T_{Ei} = (g_{int} - \gamma \epsilon_{EE}) I_{Ei} - g_{ext} I_{Ii} + \sum_{j=1}^{M} \epsilon_{ij}^{E} I_{Ej}, \tag{5}$$

$$T_{Ii} = (g_{ext} - \gamma \epsilon_{IE}) I_{Ei} - g_{int} I_{Ii} + \sum_{j=1}^{M} \epsilon_{ij}^{I} I_{Ej}, \tag{6}$$

which are composed of both intra-module and inter-module connections. By replacing the terms $I_E(t)$ and $I_I(t)$ in Eqs.(1) and (2) with T_{Ei} and T_{Ii} in Eqs.(5) and (6), a network with multiple modules is defined.

The strengths of connections are defined as

$$\epsilon_{ij}^{E} = \begin{cases} \epsilon_{EE} K_{ij} \text{ if } K_{ij} > 0 \\ 0 \qquad\quad \text{otherwise} \end{cases}, \tag{7}$$

$$\epsilon_{ij}^{I} = \epsilon_{IE} |K_{ij}|, \tag{8}$$

$$K_{ij} = \frac{1}{Ma(1-a)} \sum_{\mu=1}^{p} \eta_i^{\mu} (\eta_j^{\mu} - a), \tag{9}$$

where $\eta_i^{\mu} \in \{0, 1\}$ is the stored value in the ith module for the μth pattern, M is the number of modules, p is the number of patterns, and a is the rate of modules that store the value "1". Note that ϵ_{EE} and ϵ_{IE} scale the strengths of the inter-module connections to the excitatory and inhibitory ensembles, respectively. In the following, we set $M = 8$, $p = 3$, and $a = 0.5$.

Three patterns stored in the network of eight modules are defined as

$$\eta_i^{1} = \begin{cases} 1 \text{ if } i \leq M/2 \\ 0 \text{ otherwise} \end{cases}, \tag{10}$$

$$\eta_i^{2} = \begin{cases} 1 \text{ if } M/4 < i \leq 3M/4 \\ 0 \text{ otherwise} \end{cases}, \tag{11}$$

$$\eta_i^{3} = \begin{cases} 1 \text{ if } i \bmod 2 = 1 \\ 0 \text{ otherwise} \end{cases}. \tag{12}$$

In the following, the dynamics of the network are examined by regulating the inter-module connections ϵ_{IE}, for the fixed values of parameters $\gamma = 0.6$ and $\epsilon_{EE} = 1.25$.

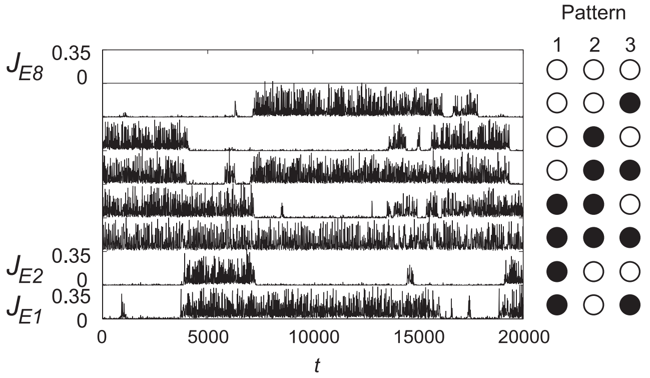

Fig. 2. Chaotic pattern alternations observed for $\epsilon_{IE} = 1.68$.

In Fig. 2, the dynamics of eight modules for $\epsilon_{IE} = 1.68$ are shown. The changes in the instantaneous firing rates J_{Ei} of the excitatory ensemble in the ith module are aligned vertically. It is observed that the retrieved pattern alters over time. The analysis of the network is performed with the Fokker-Planck equation, which does not contain noise because the limit $N_E, N_I \to \infty$ is taken. Therefore, the dynamics shown in Fig. 2 are not caused by noise but by chaos that is inherent in the network. This fact can be confirmed via analysis using Lyapunov spectra [10].

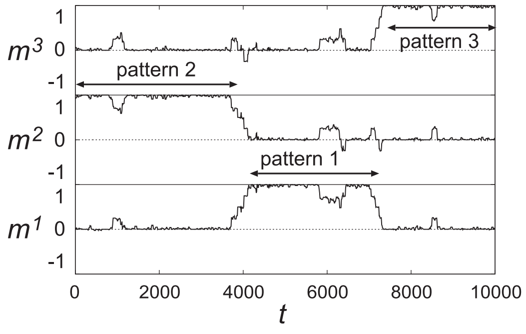

Fig. 3. Changes in three overlaps over time.

In order to investigate the retrieved pattern in the network, it is useful to define the overlap of the network with each pattern, which is similar to the inner product (detailed in Ref.[10]). The overlaps calculated from the dynamics during $0 \leq t \leq 10000$ shown in Fig.2 are shown in Fig.3. Note that m^μ takes values close to 1 when the μth pattern is retrieved.

In Figs. 2 and 3, short bursts are observed around $t \simeq 1000$, 6000, 8500, where the modules that do not store "1" in the retrieved pattern oscillate. Such patterns are referred to as mixed states in the associative memory literature. In our network with three patterns, we can observe six mixed states as shown in Fig. 4.

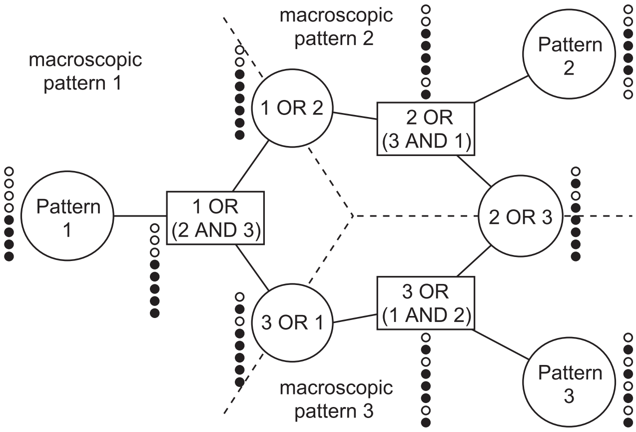

Fig. 4. The relationships among three patterns and their mixed states.

In order to incorporate the effect of the mixed state into the duration of each pattern, we defined the macroscopic duration. As shown in Fig. 3, when examining the macroscopic duration, we regard that the system retains the previously retrieved pattern even during the period when $0.5 \leq m^1, m^2, m^3 < 1$.

The macroscopic duration is based on the consideration that the mixed states represent the internal dynamics of the brain, and that these states are thus unobservable in psychological experiments. Mixed states were always unstable in the range of ϵ_{IE} in the present study, and their time-averaged duration was much shorter than those of three patterns, as shown below.

The dependence of the time-averaged values of the macroscopic duration on the inter-module connection strength ϵ_{IE} is shown in Fig. 5. All values were calculated using the durations of three patterns. We observed that the time-averaged durations diverged at the critical point $\epsilon_{IE} = \epsilon_0 \simeq 1.75$, and monotonically decreased with decreases in ϵ_{IE}. The time-averaged durations of the mixed states were always below 200 and much shorter than those of three patterns (data not shown).

Next we examine the distribution of the duration of each pattern when chaotic pattern alternations occur. The distributions of the macroscopic durations are shown in Fig. 6, in which the solid lines show the fit with the log-normal distribution. In Ref. [1], the dominant durations of binocular rivalry follow a log-normal distribution. Similarly, the distribution of the macroscopic durations in

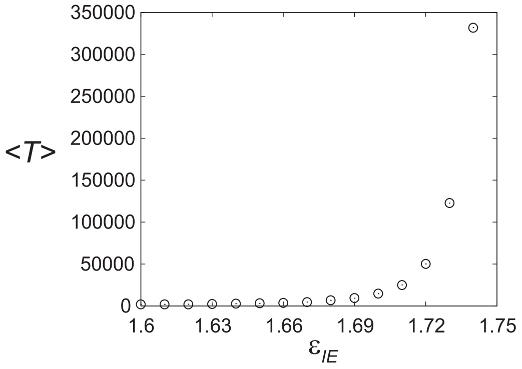

Fig. 5. The dependence of time-averaged macroscopic duration on the inter-module connection strength ϵ_{IE}.

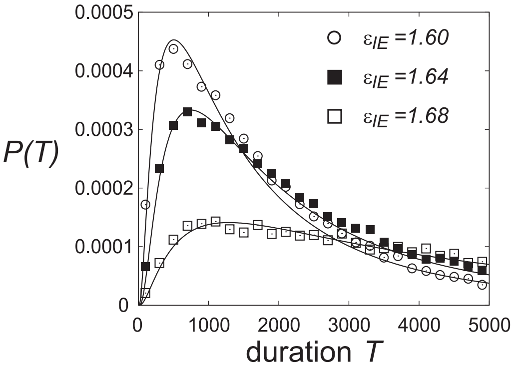

Fig. 6. The distributions of the macroscopic durations. The solid lines indicate the fit with the log-normal distribution.

our system also follows a log-normal distribution, as shown in Fig. 6. Therefore, we conclude that these macroscopic durations are appropriate as models of the dominance durations of binocular rivalry and perceptual ambiguity.

4 Conclusions

We proposed a pulse neural network that exhibits chaotic pattern alternations between three stored patterns as a model of multistable perception, which is reflected in such phenomena as binocular rivalry and perceptual ambiguity.

To measure the durations of each pattern, we introduced the macroscopic duration, which treats the mixed state as part of each pattern.

The distribution of the macroscopic durations was unimodal, following a log-normal distribution. Therefore, we conclude that the macroscopic durations of the chaotic pattern alternations can reproduce the unimodal distribution of dominance durations observed in multistable perception.

Based on these results, we propose that many pre-existing attractors in the brain might relate to the general category of multistable phenomena, such as binocular rivalry and perceptual ambiguity.

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