Biological Cybernetics manuscript No.
(will be inserted by the editor)

Detecting chaotic structures in noisy pulse trains based on interspike interval reconstruction

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Received: 7 July 2003 / Accepted: 28 February 2005 (Biological Cybernetics, vol.92, no.5 (2005) pp. 333-338.)

Abstract The nonlinear prediction method based on the interspike interval (ISI) reconstruction is applied to the ISI sequence of noisy pulse trains and the detection of the deterministic structure is performed. It is found that this method cannot discriminate between the noisy periodic pulse train and the noisy chaotic one when noise-induced pulses exist. When the noise-induced pulses are eliminated by the grouping of ISI sequence with the genetic algorithm, the chaotic structure of the chaotic firings becomes clear, and the noisy chaotic pulse train could be discriminated from the periodic one.

1 Introduction

To quantify the chaotic properties of a dynamical system, the largest Lyapunov exponent is often calculated, and, when it takes a positive value, the system has a sensitive dependence of the initial condition and its dynamics is regarded as chaotic. To calculate the Lyapunov exponent, the differential equation or the map which governs the time-evolution of the system must been known. In the field of dynamical systems, it is well-known that the state of a finite-dimensional dynamical system could be reconstructed only from a single time series using delay-coordinate vectors, and these vectors give the information about the original dynamical system. However, the simple estimation of Lyapunov exponents based on the reconstruction often leads to wrong results because the experimentally obtained time series are short and noisy.

In neural systems, a series of spikes emitted from neurons with regular or irregular time intervals can be experimentally observed. In such a system, an interspike interval sequence is thought to give some information about the neuronal networks, and the reconstruction of the dynamical system from the interspike interval sequences is performed by several authors (Theiler et al. 1992; Sauer 1994; Suzuki et al. 2000; Shinohara et al. 2002). Also in such a situation, the effects of noise often lead to wrong results, so careful treatments would be required.

In the present paper, the nonlinear prediction method (Theiler et al. 1992; Sauer 1994; Suzuki et al. 2000; Shinohara et al. 2002) based on the interspike interval (ISI) reconstruction is applied to noisy pulse trains, and the detection of the deterministic structure is performed in order to establish a method for the detection of chaotic structures in noisy pulse trains. Based on those results, the dynamical states of the system are discriminated.

The present paper is organized as follows. In Sect.2, a pulse neural network which yields noisy periodic pulse trains is defined. This model is composed of the canonical models for class 1 neurons and its mechanism of couplings has a general exponential form. Thus, similar pulse trains are expected to be observed also in biological neuronal systems. In Sect.3, the nonlinear prediction method is introduced. And the nonlinear prediction is applied to the interspike interval sequences of noisy pulse trains in Sect.4, and it is found that this method cannot discriminate between noisy periodic pulse trains and noisy chaotic ones. This is because noise-induced pulses exist. In Sect.5, the grouping of the ISI sequence with the genetic algorithm to eliminate noise-induced pulses is applied, and the discrimination between noisy periodic pulse trains and noisy chaotic ones is performed. In Sect.6, the properties of this method is summarized. Conclusions and discussions are given in the final section.

2 Pulse neural network and noisy pulse trains

In this paper, we perform the detection of chaotic structures in noisy pulse trains from a single neuron. For that purpose, we define a model to generate noisy pulse trains in this section.

Let us consider the globally connected active rotators composed of excitatory neurons $\theta_E^{(i)}$ $(i = 1, 2, \cdots, N_E)$ and inhibitory neurons $\theta_I^{(i)}$ $(i = 1, 2, \cdots, N_I)$ written as

$$\dot{\theta}_E^{(i)} = 1 - a \sin \theta_E^{(i)} + \xi_E^{(i)}(t) + I_{EE}(t) - I_{EI}(t), \quad (1)$$

$$\dot{\theta}_I^{(i)} = 1 - a \sin \theta_I^{(i)} + \xi_I^{(i)}(t) + I_{IE}(t) - I_{II}(t), \quad (2)$$

$$I_{XY}(t) = \frac{g_{XY}}{N_Y} \sum_{j=1}^{N_Y} \sum_k \frac{1}{\kappa_Y} \exp\left(-\frac{t - t_k^{(j)}}{\kappa_Y}\right), \quad (3)$$

where a is a system parameter, $I_{XY}(t)$ is the interaction from the ensemble Y to the ensemble X, and $X, Y = E$ or I (Kanamaru and Sekine 2003, 2004, 2005). Note that $\xi_E^{(i)}(t)$ and $\xi_I^{(i)}(t)$ are Gaussian white noises satisfying

$$\langle \xi_X^{(i)}(t) \xi_Y^{(j)}(t') \rangle = D \delta_{ij} \delta_{XY} \delta(t - t'), \quad (4)$$

where D is the noise intensity and δ_{ij} is Kronecker's delta.

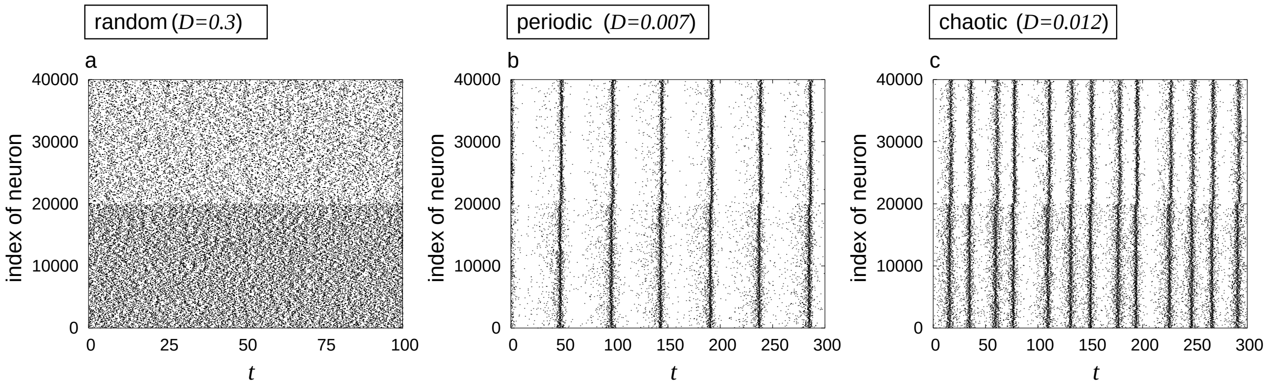

Fig. 1 The raster plots of the firing times of the system with $N_E = N_I = 20000$. **a** Random firings for $D = 0.3$, **b** synchronized periodic firings for $D = 0.007$, and **c** synchronized chaotic firings for $D = 0.012$. The connection strengths are fixed at $g_{ext} = 3.3$ and $g_{int} = 4.5$. The neurons are aligned so that the excitatory neurons are in the range $0 \leq i < 20000$ and the inhibitory neurons are in the range $20000 \leq i < 40000$.

For $a > 1$, the active rotator shows typical properties of an excitable system, namely, it has a stable equilibrium $\theta_0 \equiv \arcsin(1/a)$ and $-\sin\theta^{(i)} + 1/a$ shows a pulse-like waveform with an appropriate amount of disturbance. Note that a single active rotator can be transformed into the canonical model $\dot{\theta} = (1 - \cos\theta) + (1 + \cos\theta)r$ for class 1 neurons (Ermentrout, 1996; Izhikevich, 1999). Thus, our synaptically connected active rotators might reflect the dynamics of networks of class 1 neurons such as Connor model or Morris-Lecar model (Ermentrout, 1996).

The neurons are connected with the exponential function written by (3) where $t_k^{(j)}$ is the k-th firing time ($k = 1, 2, \cdots$) of the j-th neuron in the ensemble Y. Note that the second sum in (3) is taken over k satisfying $t > t_k^{(j)}$, and the firing time is defined as the time when $\theta_Y^{(j)}$ turns around over the value $3\pi/2$ which is the point located at the opposite side of the stable equilibrium point $\theta_0 = \arcsin(1/a) \sim \pi/2$. To reduce the parameters, we set $g_{EE} = g_{II} \equiv g_{int}$, $g_{EI} = g_{IE} \equiv g_{ext}$, $a = 1.03$, and $\kappa_E = \kappa_I = 1$.

This pulse neural network shows various firings depending on the coupling strengths and the noise intensity D. As shown in Fig.1a, when D is large, neurons fire without correlations. And, with the decrease of D, synchronized periodic firings or synchronized chaotic firings appear as shown in Figs.1b and c. In the large N_E and N_I limit, it is confirmed that the average behavior of chaotic firings in Fig.1c has positive Lyapunov exponent by analyzing the Fokker-Planck equations of the system (Kanamaru and Sekine 2003, 2004, 2005). As stated above, a single active rotator can be regarded as a canonical model for class 1 neurons, and the mechanism of couplings (3) has a general exponential form. Thus, similar dynamics might be observed also in biological neuronal systems.

Let us consider a task to detect chaotic structures in a pulse train from a single neuron in this network. If one can observe the average behavior of the system, the synchronization in the system, like Fig.1c, can be analyzed, and the detection of chaos would be easier. However, it is not obvious whether the chaotic structure in the network can be detected in a time series from a single neuron. In the following sections, we apply the nonlinear prediction based on the ISI reconstruction to pulse trains from a single neuron, and perform the detection of chaotic structures.

3 Nonlinear prediction based on ISI reconstruction

In this section, the nonlinear prediction method based on the ISI reconstruction is summarized (Sauer 1994 ; Theiler et al. 1992 ; Suzuki et al. 2000; Shinohara et al. 2002). With the k-th firing time t_k of a single neuron, the ISI is defined as

$$T_k = t_{k+1} - t_k. \tag{5}$$

Let us consider an ISI sequence $\{T_k\}$ and the delay coordinate vectors $V_j = (T_{j-m+1}, T_{j-m+2}, \cdots, T_j)$ with the reconstruction dimension m, and let L be the number of vectors in the reconstructed phase space $\boldsymbol{R}^m$. For a fixed integer j_0, we choose $l = \beta L$ ($\beta < 1$) points that are nearest to the point V_{j_0} and denote them by $V_{j_k} = (T_{j_k-m+1}, T_{j_k-m+2}, \cdots, T_{j_k})(k = 1, 2, \cdots, l)$. With $\{V_{j_k}\}$, a predictor of T_{j_0} for h steps ahead is defined as

$$p_{j_0}(h) = \frac{1}{l}\sum_{k=1}^{l} T_{j_k+h}. \tag{6}$$

With $p_{j_0}(h)$, the normalized prediction error (NPE) is defined as

$$E_{NP}(h) = \frac{\langle (p_{j_0}(h) - T_{j_0+h})^2 \rangle^{1/2}}{\langle (\langle T_{j_0} \rangle - T_{j_0+h})^2 \rangle^{1/2}}, \tag{7}$$

where $\langle \cdot \rangle$ denotes the average over j_0. When the embedded vector V_j has a structure such as periodic solutions with multiple cycles or a chaotic attractor, we say that the system has a deterministic structure. Note that a periodic solution where a neuron fires with an average interval T is regarded as a stochastic process around T, so it does not have a deterministic structure. A small value of NPE less than 1 implies that the ISI sequence has the deterministic structure behind the time series because this algorithm is based on the assumption that the dynamical structure of a finite dimensional deterministic system can be well reconstructed by the delay coordinates of ISI (Sauer 1994). However, stochastic time series with large auto-correlations can also take NPE values less than 1. Thus, we could not conclude that there is a deterministic structure only from the smallness of NPE.

To confirm the deterministic structure, the values of NPE should be compared with those of NPE for a set of surrogate data (Theiler et al. 1992). The surrogate data are new time series generated from the original time series under some null hypotheses so that the new time series preserves some statistical properties of the original

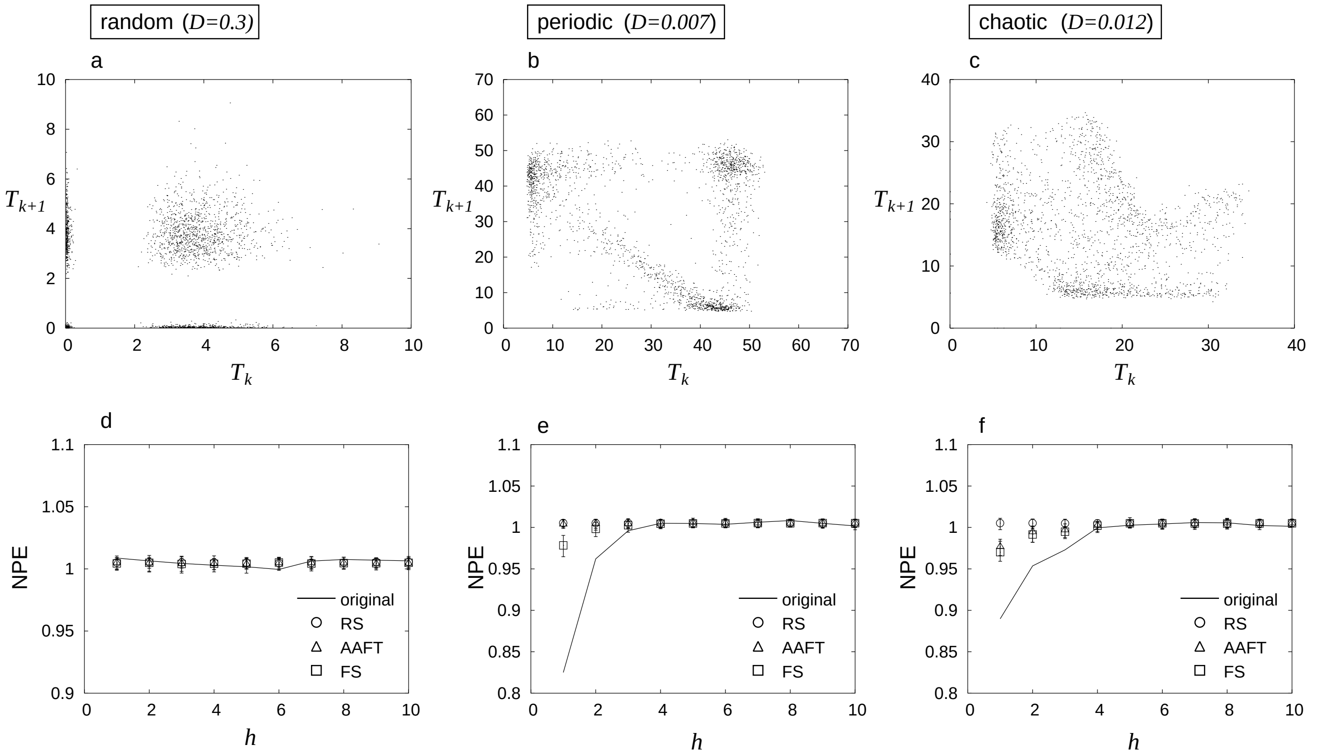

Fig. 2 **a**, **b** , **c** The return maps of the ISI sequences of the firings in Fig.1 . **c**, **d**, **e** The dependences of NPE on the prediction step h of the each firings. The error bar denotes the maximum and the minimum NPEs for 100 surrogate data.

data. In the following we use three kinds of surrogation, namely, random shuffled (RS), Fourier shuffled (FS), and amplitude adjusted Fourier transformed (AAFT) surrogate data which correspond to the null hypotheses of an independent and identically distributed random process, a linear stochastic process, and a linear stochastic process observed through a monotonic nonlinear function, respectively (Theiler et al. 1992 ; Suzuki et al. 2000; Shinohara et al. 2002). If the values of NPE for the original data are sufficiently smaller than those of NPE for the surrogate data, the null hypothesis is rejected, and it can be concluded that there is some possibility that the original time series has a deterministic structure.

4 Application of noisy pulse trains to the ISI sequence

In the following, the parameters are fixed as $m = 3$ and $\beta = 0.05$. The length of the ISI sequence $\{T_k\}$ is fixed at 2048.

The nonlinear prediction is applied to the firings in Figs.1, and the results are shown in Figs.2. Figure 2a, b, and c show the return maps of T_k obtained from a single excitatory neuron in the firings shown in Figs.1. It is observed that the random firings in Fig.1a have a mean period about 4. Note that there also exist short ISIs with $T_k \ll 1$, and they are the artifacts caused by the large noise intensity.

Figures 2d, e, and f show the dependences of NPE on the prediction step h of each firings. As shown in Fig.2d, the prediction based on the embedding is difficult for random firings, and those NPEs cannot be discriminated from those of surrogate data. Thus, this ISI sequence is regarded as a stochastic process. In contrast to the results of the random firings, NPEs for periodic firings and chaotic firings take smaller values than those of surrogate data, and they increase with the increase in the prediction step h. From these observations, it might be concluded that these ISI sequences have both deterministic structures and the sensitive dependence on the initial condition, namely, these time series are chaotic (Suzuki et al. 2000; Shinohara et al. 2002). Of course, the firings in Fig.1b are noisy periodic firings, and actually do not have a sensitive dependence on the initial condition. Thus, the above conclusions are wrong. Moreover, as stated above, periodic firings should be classified as stochastic processes because they do not have deterministic structures.

In the following, the reason why the deterministic structures are observed in the synchronized periodic firings is considered. The raster plot of the noisy periodic firings and a time series $-\sin(\theta_E^{(1)}(t))+1/a$ from the neuron 1 for $D = 0.007$ are shown in Fig.3. It is observed that the neuron 1 fires not only when synchronized firings takes place, but also when the other neurons are almost silent. Such firings are indicated by vertical arrows in Fig.3, and we call them noise-induced pulses in the following.

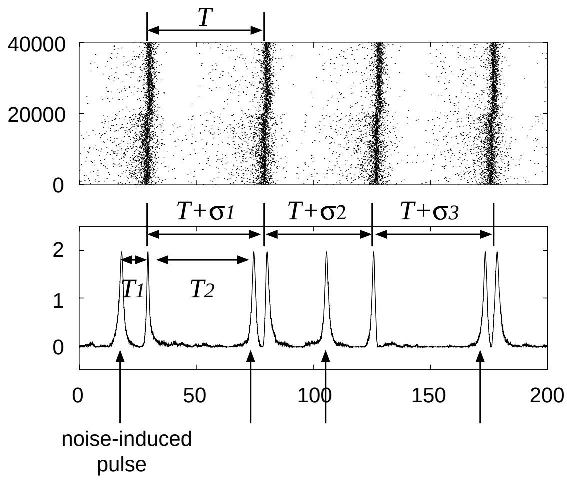

Fig. 3 The raster plot of the noisy periodic firings and a time series $-\sin(\theta_E^{(1)}(t)) + 1/a$ for $D = 0.007$.

When the period of synchronized firings is T, the corresponding ISIs in a single neuron takes $T + \sigma_i$ where σ_i is a random variable as shown in Fig.3. However, the actual ISI sequence does not necessarily take such values because there also exist noise-induced pulses. Actually, T_k tends to suffice the relation $T_k + T_{k+1} \sim T$ as shown in Fig.3. For the noisy periodic firings in Fig.2b, it is observed that $T \sim 45$ and many data suffice the relation $T_k + T_{k+1} \sim 45$. When such a structure exists, the prediction becomes easy, and NPEs take small values. Moreover, the long-term prediction is difficult because there exists noise in the system. Thus, NPE increases with the increase of the prediction step h. For the above reasons, the synchronized periodic firings were regarded as a deterministic time series with a sensitive dependence on the initial condition by mistake.

On the other hand, even for the chaotic ISI sequence shown in Fig.2c, fine structures cannot be observed. Thus, it is not clear whether the observed ISI sequence really has chaotic properties. This is because the noise-induced pulses exist also in the chaotic ISI sequence.

In the following, we propose a method to eliminate the noise-induced pulses, and discriminate between the periodic pulse train and the chaotic one.

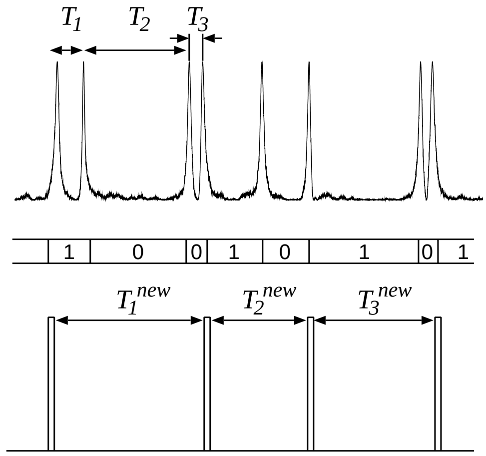

Fig. 4 The coding to the genes and a new ISI sequence.

5 Grouping the ISI sequence with the genetic algorithm

Let us consider an ISI sequence $\{T_k\}_{k=1}^{K}$ and group it into M groups $(T_1, \cdots, T_{k_1})$, $(T_{k_1+1}, \cdots, T_{k_2}) \cdots (T_{k_{M-1}+1}, \cdots, T_{k_M})$ where $k_0 = 0$ and $k_M = K$. With this grouping, a new ISI sequence T_n^{new} is defined as

$$T_n^{new} = \sum_{k=k_{n-1}+1}^{k_n} T_k. \qquad (8)$$

If the grouping which eliminates the noise-induced pulses is applied to a noisy periodic pulse train, the ISIs of the new sequence would take the values which are close to the original period. To realize such a grouping, we minimize the standard deviation σ_T of T_n^{new} with the genetic algorithm (GA) (Davis 1996). Let us consider the coding to the gene where it takes 1 for T_{k_n+1} and takes 0 otherwise for the original sequence $\{T_k\}$ as shown in Fig.4. When such a coding is determined, the new ISI sequence T_n^{new} is obtained, and σ_T can be calculated. If $1/\sigma_T$ is maximized with GA, the optimal grouping would be realized. Note that the number M of the groups is not fixed, so it is variable during the process of GA.

The mean value T and the standard deviation σ_T of T_n^{new} during GA for the ISI sequence of the noisy periodic firings are shown in Fig.5. The genes are initialized randomly. It is observed that σ_T converges to a small value. Similarly, a new ISI sequence is also obtained for the chaotic firings.

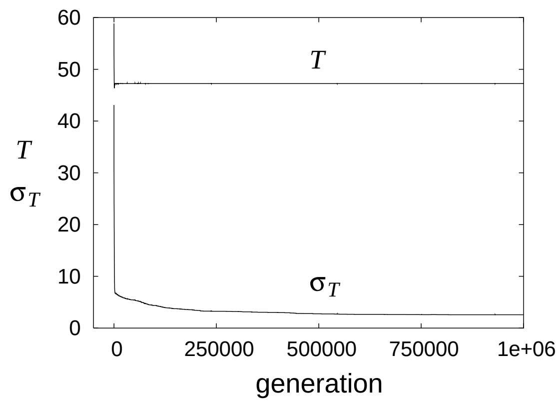

Fig. 5 The mean value T and the standard deviation σ_T of T_n^{new} during GA for the ISI sequence of noisy periodic firings.

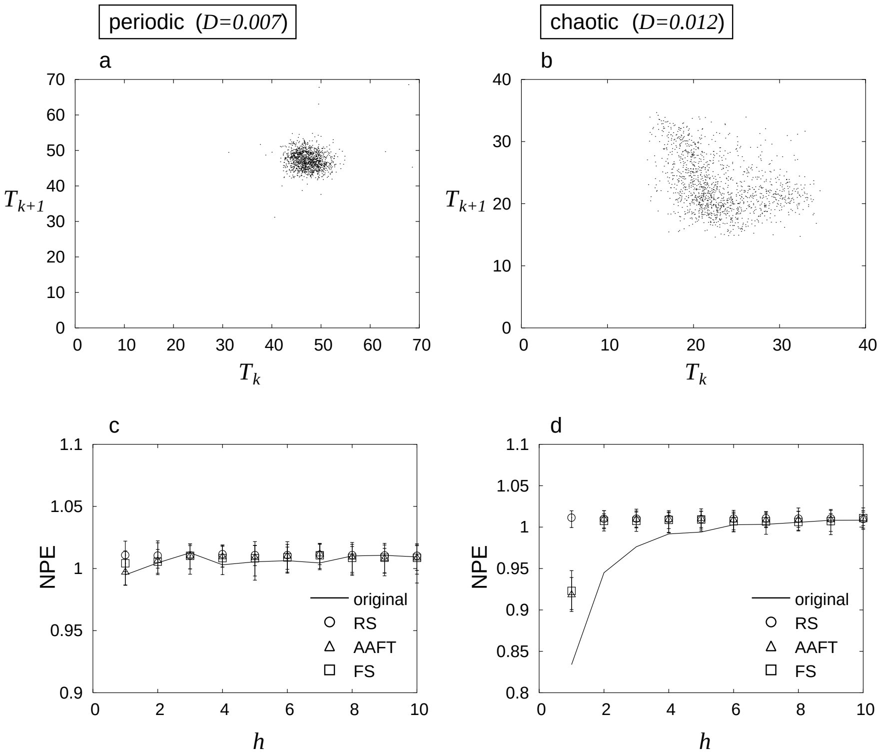

Fig. 6 The results of nonlinear prediction applied to the new ISI sequences T_n^{new}. **a**, **b** The return maps of the ISI sequences of **a** periodic firings and **b** chaotic firings. **c**, **d** The dependences of NPE on the prediction step h of **c** the periodic firings and **d** the chaotic firings. The error bar denotes the maximum and the minimum NPEs for 100 surrogate data.

The results of the nonlinear prediction applied to the new ISI sequences T_n^{new} are shown in Fig.6. As shown in Fig.6a, for the new ISI sequence of periodic firings, the noise-induced pulses are eliminated, and the periodicity of the ISI sequence is restored. And, as shown in Fig.6c, its NPEs cannot be discriminated from those of surrogate data. Thus, it can be regarded as a stochastic process. For the new ISI sequence of chaotic firings, the return map shows a clear parabolic structure as shown in Fig.6b. This structure reflects the chaotic structure of the synchronized firings in Fig.1c. Namely, the noise-induced pulses are eliminated by the grouping, and the chaotic structure is restored. As shown in Fig.6d, its NPEs can be easily discriminated from those of surrogate data, and the sensitive dependence on the initial condition becomes more clear.

6 Properties of the proposed method

In the previous section, we proposed a grouping method of ISI sequences, and performed detections of chaotic structures in noisy pulse trains based on the ISI reconstruction. To detect a chaotic structure in an ISI sequence, both the grouping and the reconstruction must be successful.

Empirically, the grouping seems to be almost always successful when low-dimensional structures such as Figs.6a and b exist. This would be because the variance of an ISI sequence takes small values when it is confined in some local structure.

On the other hand, the prediction based on the reconstruction does not always give successful results, and such an example is shown in Fig.7. Although the used ISI sequence is obtained from chaotic synchronized firings and the grouping is successful, the prediction based on the reconstruction could not distinguish this ISI sequence from stochastic processes. This is because the chaotic structure is hidden in noise as shown in Fig.7a, not because the grouping failed. In other words, the fluctuation σ_i in Fig.3 is too large and a chaotic structure like Fig.6b is lost. Thus, it could be concluded that this prediction based on the reconstruction gives reasonable results only when the observed neuron preserves the precision of the spike timing to some extent and when noise is small enough not to destroy the original chaotic structure. In other words, this method is suitable for the ISI sequences from neurons whose information is coded in the timings of spikes. On the other hand, the ISI sequence used in Fig.7 does not preserve the timings of the chaotic synchronization, and the chaotic property is coded in the firing rate of the network. Thus, the detection of the chaotic structure fails.

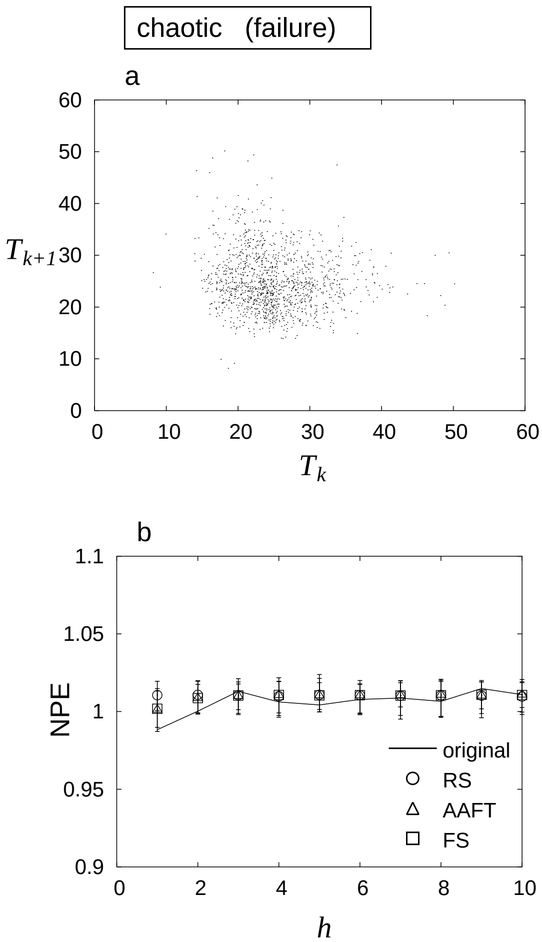

Fig. 7 The failure of the detection of the chaotic structure for $a = 1.05$, $D = 0.016$, $g_{ext} = 3.4$, and $g_{int} = 4$. **a** The return maps of the ISI sequences after the grouping. **b** The dependences of NPE on the prediction step h. Although the grouping is successful, the prediction could not distinguish this ISI sequence from stochastic processes.

7 Conclusions and discussions

To establish a method for the detection of chaotic structures in noisy pulse trains, the nonlinear prediction method based on the ISI reconstruction is applied to the noisy ISI sequence, and the detection of the deterministic structure is performed.

For random firings, the NPEs for the original data cannot be discriminated from those of surrogate data. Thus, those data are regarded as stochastic precesses.

For the synchronized periodic firings and the synchronized chaotic firings, their NPEs imply the deterministic structures and the sensitive dependence on the initial condition, but they are wrong results caused by the noise-induced pulses.

When the nonlinear prediction method is applied after the elimination of noise-induced pulses by the grouping with the GA, the synchronized periodic firings and the synchronized chaotic firings are discriminated. Particularly, the chaotic structure of the chaotic firings becomes clear after the grouping.

The flow of the nonlinear prediction with the grouping is shown in Fig.8. With the grouping, we can discriminate between the noisy periodic sequence and the noisy chaotic sequence. This method gives reasonable results only when the observed neuron preserves the precision of the spike timing to some extent and when noise is small enough not to destroy the original chaotic structure. Thus, this method is suitable for the ISI sequences from neurons whose information is coded in the timings of spikes.

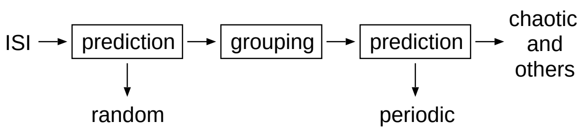

Fig. 8 The flow of the nonlinear prediction with the grouping. With the grouping, noisy periodic pulse trains and chaotic pulse trains are discriminated.

Note that the ISI sequences which are not regarded as periodic after the second prediction are not always chaotic. For example, a periodic sequence with two or more cycles is not regarded as periodic with this method, but of course they are not chaotic. Thus, more careful analyses will be required for the experimentally obtained time series.

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